## Measures of Association

## Association

Statistical dependence between two or more events, characteristics, or variables

An association is present when the probability of occurrence of an event or characteristic, or the quantity of a variable, depends upon the occurrence of one or more other characteristics, or the quantity of one or more variables

## What is RISK?



## What is RISK?

probability of developing a condition over a defined period of time
\# of people developing disease in time period
\# of disease-free people at start of period

$$
\text { \# of people developing TB in } 5 \text { years }
$$

\# of people free of TB at start of 5 yr period

## 1-year follow-up for the development of CHD

|  |  | CHD |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | + | - | Total |
| Smoking | + | 84 | 2916 | 3000 |
|  | - | 87 | 4913 | 5000 |

Incidence in exposed ${ }^{+}=84 / 3000=28.0 / 1000$ Incidence in exposed ${ }^{-}=87 / 5000=17.4 / 1000$

|  |  | CHD |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | - | Total |  |
| Smoking | + | 84 | 2916 | 3000 |
|  | - | 87 | 4913 | 5000 |

## Risk Difference/Attributable Risk

Incidence in exposed ${ }^{+}=84 / 3000=28.0 / 1000$ Incidence in exposed ${ }^{-}=87 / 5000=17.4 / 1000$

$$
R D=28.0-17.4=10.6 / 1000
$$

Among every 1000 smokers, 10.6 more cases of CHD develop than among 1000 non-smokers

## Risk Difference

- RD $>0=$ positive association
- $\mathrm{RD}=0=\mathrm{no}$ association
- RD < 0 = negative association


## Relative Risk

Probability of developing disease if risk factor is present

Probability of developing disease if risk factor is absent

Incidence of stroke in hypertensives
Incidence of stroke in normotensives

|  |  | CHD |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | - | Total |  |
| Smoking | + | 84 | 2916 | 3000 |
|  | - | 87 | 4913 | 5000 |

## Relative Risk or Risk Ratio

Incidence in exposed ${ }^{+}=84 / 3000=28.0 / 1000$ Incidence in exposed ${ }^{-}=87 / 5000=17.4 / 1000$

$$
R R=28.0 \div 17.4=1.61
$$

Risk for CHD among smokers is 1.6 times higher than among non-smokers

## Relative Risk

- RR > 1 = positive association
- $R R=1=n o$ association
- $R R<1$ = negative association

RR tells us about the strength of association between exposure and outcome but not about the magnitude of absolute risk (incidence)

## Odds:

- The ratio of the probability of occurrence of an event to that of Odds Ratio (or Relative Odds)
- The ratio of the probability that something will occur, to the probability that it will not occur (Abramson, MSoD)


## "Odds is understood intuitively only by statisticians and professional gamblers"



## Odds Ratio (OR)

outcome
exposure


## Odds Ratio (OR)

## outcome

exposure


## Odds Ratio (OR)

| outcome |  |
| :---: | :---: |
| exposurea b <br> $c$ $d$ |  |

## Prospective study (Disease OR)

Ratio of odds of outcome in exposed ( $\mathrm{a} / \mathrm{b}$ ) to the odds of outcome in the unexposed (c/d):

$$
\frac{a / b}{c / d}=\frac{a d}{b c}
$$

|  |  | CHD |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | - | Total |  |
| Smoking | + | 84 | 2916 | 3000 |
|  | - | 87 | 4913 | 5000 |

## Odds Ratio

odds in exposed ${ }^{+}=84 / 2916=28.8 / 1000$ odds in exposed ${ }^{-}=87 / 4913=17.7 / 1000$

$$
\mathrm{OR}=28.8 \div 17.7=1.63
$$

ODDS for CHD among smokers is 1.63 times higher than among non-smokers

## Odds Ratio

- OR > 1 = positive association
- $O R=1$ = no association
- $\mathrm{OR}<1$ = negative association


## $O R \cong R R$ when...

1. Outcome is rare because then:

$$
b \cong(a+b) \text { and } d \cong(c+d)
$$

2. Cases are representative of all cases in the population re. exposure history
3. Controls representative of all non-cases in the population re. exposure history
outcome

exposure | $a$ | $b$ | $a+b$ |
| :---: | :---: | :---: |
| $c$ | $d$ | $c+d$ |



## OR in a retrospective study

outcome
exposure


## Odds Ratio (OR)

outcome
exposure


## outcome

exposure

| $\mathbf{a}$ | $\mathbf{b}$ |
| :--- | :--- |
| $\mathbf{c}$ | $\mathbf{d}$ |

Retrospective study (Exposure OR):
Ratio of odds of exposure in cases (a/c) to the odds of exposure in controls (b/d):

$$
\frac{a / c}{b / d}=\frac{a d}{b c}
$$

## RR \& OR

- OR is a useful measure of association in and of itself, in both case-control and prospective studies
- In a cohort study, RR can be calculated directly
- In a case-control study, RR cannot be calculated directly, so OR is used as an estimate of RR when the risk of disease is low


## Advantages of Odds Ratio

- Suitable for case-control studies
- Identical in both directions
- Basis for logistic regression analyses


## Now you try it!

| History of <br> sexual <br> abuse | Suicide attempted |  |
| :---: | :---: | :---: |
|  | 14 | No |
| No | 49 | 149 |

## Deykin \& Buka (1994) AJPH 84:634-9

| History of <br> sexual <br> abuse | Suicide attempted |  |  |
| :---: | :---: | ---: | :---: |
|  | No | Total |  |
| Yes | 14 | 9 | 23 |
| No | 49 | 149 | 198 |

$$
\begin{gathered}
R_{\text {exp }+}=14 / 23=60.9 \% \\
R_{\text {exp- }}=49 / 198=24.7 \% \\
R R=60.9 / 24.7=2.46
\end{gathered}
$$

| History of <br> sexual <br> abuse | Suicide attempted |  |  |
| :---: | :---: | :---: | :---: |
|  | No | Total |  |
| No | 49 | 9 | 23 |

$$
R R=\frac{14 / 23}{49 / 198}=2.46
$$

| History of <br> sexual <br> abuse | Suicide attempted |  |
| :---: | :---: | ---: |
|  | 14 | No |
| No | 49 | 149 |

$$
O R=\frac{14 / 9}{49 / 149}=4.73
$$

## Question: Why do the OR and RR differ?

## Answer:

## high incidence

(61\% in exposed, $25 \%$ in unexposed)
therefore $b \neq a+b$ and $d \neq c+d$

